Exercise 6B

1 a $\sin 135° = +\sin 45°$

(135° is in the second quadrant at 45° to the horizontal.)

So $\sin 135° = \frac{\sqrt{2}}{2}$

b $\sin(-60)° = -\sin 60°$

(–60° is in the fourth quadrant at 60° to the horizontal.)

So $\sin(-60°) = -\frac{\sqrt{3}}{2}$

c $\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

($\sin\left(\frac{11\pi}{6}\right)$ is in the fourth quadrant, at $\left(\frac{\pi}{6}\right)$ to the horizontal.)

So $\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

d $\sin\left(\frac{7\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$

($\sin\left(\frac{7\pi}{3}\right)$ is in the first quadrant, at $\left(\frac{\pi}{3}\right)$ to the horizontal.)

So $\sin\left(\frac{7\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

e $\sin(-300°) = +\sin 60°$

(–300° is in the first quadrant at 60° to the horizontal.)

So $\sin(-300°) = \frac{\sqrt{3}}{2}$

f $\cos 120° = -\cos 60°$

(120° is in the second quadrant at 60° to the horizontal.)

So $\cos 120° = -\frac{1}{2}$

1 g $\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

($\left(\frac{5\pi}{3}\right)$ is in the fourth quadrant, at $\left(\frac{\pi}{3}\right)$ to the horizontal.)

So $\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

h $\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$

($\left(\frac{5\pi}{4}\right)$ is in the third quadrant, at $\left(\frac{\pi}{4}\right)$ to the horizontal.)

So $\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

i $\cos\left(-\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$

($\left(-\frac{7\pi}{6}\right)$ is in the second quadrant, at $\left(\frac{\pi}{6}\right)$ to the horizontal.)

So $\cos\left(-\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

j $\cos 495° = -\cos 45°$

(495° is in the second quadrant at 45° to the horizontal.)

So $\cos 495° = -\frac{\sqrt{2}}{2}$

k $\tan 135° = -\tan 45°$

(135° is in the second quadrant at 45° to the horizontal.)

So $\tan 135° = -1$

l $\tan(-225°) = -\tan 45°$

(–225° is in the second quadrant at 45° to the horizontal.)

So $\tan(-225°) = -1$

1 m $\tan\left(\frac{7\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right)$

($\left(\frac{7\pi}{6}\right)$ is in the third quadrant at $\left(\frac{\pi}{6}\right)$ to the horizontal.)

So $\tan\left(\frac{7\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

n $\tan 300° = -\tan 60°$
(300° is in the fourth quadrant at 60° to the horizontal.)

So $\tan 300° = -\sqrt{3}$

o $\tan\left(-\frac{2\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right)$

($\left(-\frac{2\pi}{3}\right)$ is in the third quadrant at $\left(\frac{\pi}{3}\right)$ to the horizontal.)

So $\tan\left(-\frac{2\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

Challenge

a i $\tan 30° = \frac{1}{CE}$

$$CE = \frac{1}{\tan 30°} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

ii Using Pythagoras' theorem

$$CD^2 = 1^2 + \sqrt{3}^2$$
$$CD = \sqrt{1+3}$$
$$CD = 2$$

iii Using Pythagoras' theorem on the isosceles triangle ABC

$$AB^2 + BC^2 = (1+\sqrt{3})^2$$
$AB = BC$ so $BC^2 + BC^2 = (1+\sqrt{3})^2$
$$2BC^2 = 4 + 2\sqrt{3}$$
$$BC^2 = 2 + \sqrt{3}$$
$$BC = \sqrt{2+\sqrt{3}}$$

iv $DB = AB - AD$
Using Pythagoras' theorem

$$AD = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$DB = \sqrt{2+\sqrt{3}} - \sqrt{2}$$

b Angle $BCD = 45° - 30° = 15°$

c i $\sin 15° = \frac{DB}{CD}$

$$= \frac{\sqrt{2+\sqrt{3}} - \sqrt{2}}{2}$$

ii $\cos 15° = \frac{BC}{CD} = \frac{\sqrt{2+\sqrt{3}}}{2}$